

Properties of Logarithms Section 0.5 Calculus

Warm-up: Simplify.

- | | | |
|--------------------|-----------------------------|-------------------|
| 1. $x^4 \cdot x^7$ | 2. $\frac{x^9}{x^6}$ | 3. $(x^5)^8$ |
| 4. $2^{x+1} = 8$ | 5. $3^{2x-3} = \frac{1}{3}$ | 6. $8^{5-2x} = 1$ |

Notes about Logarithms

1. What if you cannot make the bases the same? Consider: $7^{2x} = 12$.

How do you solve an equation where the unknown variable is in the exponent? _____.

2. A logarithm is an _____.

Evaluating Logarithms

- | | | | |
|---------------|--------------------------|------------------------|-------------------------|
| a) $\log_2 8$ | b) $\log_5 \frac{1}{25}$ | c) $5\log_6 1$ | d) $\log_7 0$ |
| e) $\log 100$ | f) $\log 1$ | g) $5\log \sqrt{10}$ | h) $\log 10^{\sqrt{5}}$ |
| i) $\ln e^4$ | j) $9\ln 1$ | k) $\ln \frac{1}{e^3}$ | l) $e^{\ln 5}$ |

Properties of Logarithms

- | | | | |
|-----------------------|-----------------------|-------------------------|---------------------------|
| 1. $\log_b b =$ _____ | 2. $\log_b 1 =$ _____ | 3. $\log_b b^a =$ _____ | 4. $b^{\log_b n} =$ _____ |
|-----------------------|-----------------------|-------------------------|---------------------------|

Change of Base Formulas

Example 1: Evaluate $\log_4 25$

Base	Base 10	Base e
Formula		

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Example 2: Evaluate $\log_3 16$ using

a) the change of base formula with common logs (round to 4 decimal places).

b) the change of base formula with natural logs (round to 4 decimal places).

Properties of Logarithms

	Logarithm with Base a	Natural Logarithm
Product Property	$\log_a (uv) =$	$\ln(uv) =$
Quotient Property	$\log_a \left(\frac{u}{v}\right) =$	$\ln\left(\frac{u}{v}\right) =$
Power Property	$\log_a u^n =$	$\ln u^n =$

Example 3: Write each logarithm in terms on $\ln 2$ and $\ln 3$.

a) $\ln 6$

b) $\ln \frac{2}{27}$

Practice Problem 2: Write each logarithm in terms of $\ln 2$ and $\ln 5$.

a) $\ln 10$

b) $\ln \frac{5}{32}$

Example 4: Use the properties of logs to expand each expression.

a) $\log_4 5x^3y$

b) $\ln \frac{\sqrt{3x-5}}{7}$

Practice Problem 3: Use the properties of logs to expand each expression.

a) $\log 3x^2y$

b) $\ln \frac{\sqrt{4x+1}}{8}$

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Example 5: Use the properties of logs to condense each expression to a single log (or ln).

a) $\frac{1}{2} \log_{10} x + 3 \log_{10} (x+1)$

b) $2 \ln(x+2) - \ln x$

c) $\frac{1}{3} [\log_2 x + \log_2 (x-4)]$

Example 6

If $a = \log_2 6$ and $b = \log_2 10$, express the following in terms of a and b .

a) $\log_2 24$

b) $\log_2 600$

c) $\log_2 \sqrt[4]{10}$

Solving Logarithmic Equations

1. Logarithmic and Exponentials Functions are _____.
2. Solve log equations (undo the log) by _____.
3. Solve exponential equations (undo the exponent) by _____.

Examples of Solving Log and Exponential Equations

1. $\log x = \sqrt{2}$

2. $\ln(x+1) = 5$

3. $5^x = 7$

4. $7^{2x} = 12$

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Class Work

Evaluate the logarithm using the change of base formula. Round to three decimals.

1. $\log_7 4$

2. $\log_{20} 175$

Evaluate the logarithm using the properties of logs, given

$$\log_b 2 \approx 0.3562$$

$$\log_b 3 \approx 0.5646$$

$$\log_b 5 \approx 0.8271$$

3. $\log_b 30$

4. $\log_b \frac{16}{25}$

Use the properties of logs to expand the expression.

5. $\ln \frac{xy}{z}$

6. $\ln \sqrt{\frac{x^2}{y^3}}$

Use the properties of logs to condense the expression to a single log or ln.

7. $2 \ln x + \ln(x+1)$

8. $3 \ln x + 2 \ln y - 4 \ln z$

Use the properties of logs to rewrite the expression in terms of r , s , and t given

$$r = \ln a$$

$$s = \ln b$$

$$t = \ln c$$

9. $\ln a^2 \sqrt{bc}$

10. $\ln \frac{b}{a^3 c}$

Solve for x .

11. $3e^{-2x} = 5$

12. $\log_3(3^x) = 7$